

Population Projection Model in An Ecological Framework.

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Abstract

This work considered an important aspect of the development of statistical methods used to evaluate ecological populations. However, this work focused on the ecological aspect of the environment, we considered population growth models with adjustments depicting various changes in the ecological system and we also proposed models to monitor extinction of various species of organisms. In particular, we want to know if an important species of animals is going into extinction. As a result of climate change, changes in habitat(Degradation of Habitat) and other pressures on the environment, there is extensive interest in studying wildlife populations.

To be able to understand these populations, reliable representations of the underlying processes are required by using statistical models defined by parameters that describe different characteristics, such as the survival rate of an animal or the proportion of all the animals in a population seen and recorded. Each of these characteristics can be based on the ages or locations of animals and the year or month that measurements of the population took place.

We incorporate two parameters into growth model indicating change that could cause decline in population, time derivatives and time for the model to double in size was also derived.

Keywords: Ecological Population, Species, Extinction, Climate Change, Degradation of Habitat, Man Population Increase, Statistical Methods.

1. Introduction

This present work will focus on the ecological aspect of the environment, it will consider population growth model with adjustments depicting/indicating various change in the ecological system and we could also propose a models to monitor extinction of various species of organism. All these have lead into development of environmental statistics as an area of specialization. This area of statistics has been developed over the past few years in response to increasing concerns among individuals, organizations, and government for protecting the environment. Many population of some spices are gradually going to extinction as a result of man's activities and degradation of habitat.

Moreover, in other to study how to bring declining population back to stability, the numbers of species of plants and animals that are going into extinction due to climatic change and degradation of habitats of man's activities have to be considered. However, degradation of the natural

environment is a worldwide problem, this term is used specifically to refer to damages caused by human activities rather than natural ones, and human activities can indirectly contribute to environmental changes that may accelerate the speed of degradation. The more damaged the Habitat becomes, the less it can support life. This can cause degradation to speed up, as plants and animals that would normally help restore the soil are unable to survive.

The process of restoring the habitat that has become degraded is known as remediation. In remediation, we identify the causes of the degradation and try various alternatives to reverse it. Usually remediation takes time, as scientists, we want to encourage the land and ecosystem to rebuild and become stable again rather than enacting a quick fix. In some cases, habitats are too badly degraded for remediation to be effective, forcing populations that relied on the habitat to relocate in order to access new resources.

Increase in man's population and degradation of habitats have caused a decline in the number of many species of animals. This decline may lead to near extinction of useful species, however this extinction should be vehemently prevented by man, and it has become increasingly important in attempting to preserve populations of rare or endangered species. But for man to make decisions for any species, he must estimate the population's response to various alternatives to know what must have been causing the decline in population.

In Abigail et.al (2009) they noted that there are two kinds of uncertainty or variation, which is thought to be of particular importance to population growth and have been identified as demographic variation and environmental variation. Both of these processes have a strong effect on λ and they contribute to a population's risk of extinction; Land (2002). They were able to identify another potential and unpredictable cause of population depletion is catastrophes, these are large environmental perturbations that produce sudden major reductions in population size. Although there is no strict definition of what constitutes a catastrophe, the term is generally restricted to events that result in a decrease in population size of at least 50%. Catastrophes include physical factors such as hurricanes, freezes and droughts, biological factors such as epidemics or invasion by a new competitor or predator; Abigail et.al (2009).

In Begon and Mortimer (1986) they noted that considerable attention should be paid to the manner in which plant and animal populations are being exploited for the benefit of mankind. However, many of the world's plant and animal species have been hunted almost to extinction and others, such as the North American passenger pigeon; have been driven to extinction as a direct consequence of over exploitation see Reynolds (2001).

1.1 Objectives of the Study

It is aimed at introducing population models and monitoring extinction in an ecological framework.

Specifically the objectives are

- To monitor extinction and investigate the extent of endangering.
- To know if there is a decline in the population of some species and project the population size into the future.
- To introduce population growth models in an ecological framework.

1.2 Model 1

Assuming you own a piece of land and on that land there is a large cage and you are given four rabbits as a present and these rabbits were placed in this cage, sometimes ago in May 1997 and before the year runs out you find out that half of the rabbits survives to gives birth to there new ones in the next year May 1998, suppose now you have eighty rabbits in your cage, and half of the numbers in 1998 did not make it to 1999 May, but the numbers of your rabbits increase to be sixteen and half made it to the next year and now you have thirty two rabbits in your cage, and in this years you are able to identifier cause of death different from normal death as climatic change .

- Where t is the number of time units (years in our model).
- N_0 is the size of our population at the beginning, λ Is the per capita rate of increase over the specified time interval.
- N_t Is the predicted size of the population after time t .
- θ_1 the probability that some species will not survive due to climatic change
 $0 < \theta_1 < 1$.
- $N_{1997} = \text{year 1997}$

Now we have a problem on how to describe the pattern of growth and what the population size will be in the future, and we will like to know the rate of increase in each year.

Rate of increase for each year.

$$\frac{N_{1998}}{\theta_1 N_{1997}} = \frac{8}{4 \cdot 0.5} = 4$$

$$\frac{N_{1999}}{\theta_1 N_{1998}} = \frac{16}{8 \cdot 0.5} = 4$$

$$\lambda = 4$$

Where four (4) is the rate of change.

In generalizing the principle, we have

$$N_{t+1} = 4N_t\theta_1$$

$$\frac{N_{t+1}}{N_t\theta_1} = 4$$

The above equations let us describe the rate of change in population size from year to the next year, with probability that not all the organisms will survive to the next year. And we will like to predict population size for 30 years.

Let us start with one year and try to describe how it will be from there,

$$N_0 = 4$$

$$N_{1998} = 0.5 * N_0 * 4 = 8$$

$$N_{1999} = 0.5^2 * N_0 * 4^2 = 16$$

$$N_{1998} = 0.5^3 * N_0 * 4^3 = 32$$

Then, the general rule will be,

$$N_t = N_0\theta^t\lambda^t$$

1.3 Continuous Exponential Growth

Even though many organisms (plant, animals) are modelled well with discrete growth models many populations are poorly represented by discrete growth models. These populations are often modelled as continuously growing populations. Such models take advantage of simple calculus, the mathematics of rates (Henry and Steven 2009).

Whereas geometric growth is proportional change in a population over a specified finite time interval, exponential growth is proportional instantaneous change over an instant. A long time ago, some people realized that geometric growth depends on how often you think a step of increase occur. Suppose λ increase by 100%

Then

$$N_t = N_0\theta^t\lambda^t$$

$$\lambda = 1 + rd$$

So the discrete growth is $rd = 1$. You can think a step of growth occur over two time step. Therefore when

$$\lambda^2$$

$$N_t = N_0 \theta \left(1 + \frac{1}{2}\right)^2$$

$$N_t = N_0 \theta (1 + 0.5)^2$$

What if we keep on increasing the number of time steps

$$N_t = N_0 \theta \left(1 + \frac{rd}{n}\right)^n$$

What will happen to $\left(1 + \frac{rd}{n}\right)^n$ as n goes to infinite

$$\lim_{n \rightarrow \infty} \left(1 + \frac{rd}{n}\right)^n$$

This above has a simple mathematical expression and it is called exponential (e). Recall that e is one of the magic numbers in mathematics that keeps popping up everywhere. In this context, we find that for large values of n

$$\lim_{n \rightarrow \infty} \left(1 + \frac{rd}{n}\right)^n = e^r$$

This means when a population is growing geometrically with finite small time step, then we called it the population growing exponentially. We represent this as

$$N_t = N_0 \theta^t e^{rt}$$

Where r is intrinsic rate of increase

1.4 Deriving the Time Derivatives}

We can differentiate $N_t = N_0 \theta^t e^{rt}$ with respect to time to get the differential equation for instantaneous population growth rate. By using chain rule, the derivatives of a product of two are the sum of the product of the derivative of one times the other original term.

N_0 is a constant

$$U = \theta^t$$

$$V = e^{rt}$$

$$\frac{du}{dt} = \theta^t \ln \theta$$

$$\frac{dy}{dt} = re^{rt}$$

$$N_t = N_0 \theta^t e^{rt}$$

$$\frac{dN}{dt} = N_0 \left[\theta^t \frac{d}{dt} e^{rt} + e^{rt} \frac{d}{dt} \theta^t \right]$$

$$\frac{dN}{dt} = N_0 e^{rt} \theta^t [r + \ln \theta]$$

$$\frac{dN}{dt} = N[r + \ln \theta]$$

This is the time derivative for growth model given above.

1.5 Doubling Time

The doubling time of a population is the time required for a population to double in size. By double time we mean that $N_t = 2N_0$

$$N_t = N_0 \theta^t e^{rt}$$

The time it will take the population to double in size

$$2N_0 = N_0 \theta^t e^{rt}$$

$$2 = \theta^t e^{rt}$$

Take natural logarithm of both sides

$$\ln 2 = t \ln \theta + tr$$

$$\ln 2 = t(\ln \theta + r)$$

Divide both sides by $t(\ln \theta + r)$

$$t = \frac{\ln 2}{t(\ln \theta + r)}$$

This is the time it will take the population to double in size

1.6 Model 2

Assuming that you own a piece of land and on that land there is a garden, you are given four birds as a present, and placed this birds in this garden of yours, in May 1997 and before the year runs out you find out that some dead and we want to know the cause of death, that about 10% of the birds dead due to climatic condition to year May 1998, suppose you now have eighty birds in your garden, and about 10% of the numbers in 1998 did not make it to 1999 May, but the numbers of your birds increase to be sixteen and about 10% did not make it to the next year and now you have thirty two birds in your garden. And in each year there is another cause of death, that about 80% of the birds survivors from one year to another due to degradation of habitats through man's activities. You are able to identifier two different cause of death and percentage of survive due to each death, 90% survivals from the population due to climatic change, 80% survivals from the population due to degradation (man's activities) of habitat.

Where t is the number of time units (years in our model).

N_0 Is the size of our population at the beginning?

λ Is the finite rate of increase, it is the per capita growth of a population, if population is growing geometrically.

N_t Is the predicted size of the population after time t .

θ_1 The probability that not all the population will make the transition to the next stage due to changes in ecosystem (climatic condition), $0 < \theta_1 < 1$

θ_2 Probability that degradation (from man) will affect the size of the population, $0 < \theta_2 < 1$

Now we have a problem on how to describe the pattern of growth and what the population size will be in future and we will like to know the rate of increase in each year.

$$N_{1997} = N_0 = 4$$

Rate of change

$$\frac{N_{1998}}{\theta_1 \theta_2 N_{1997}} = \frac{8}{4 \cdot 0.8 \cdot 0.9} = 2.777778$$

$$\frac{N_{1999}}{\theta_1 \theta_2 N_{1998}} = \frac{16}{8 \cdot 0.8 \cdot 0.9} = 2.777778$$

$$\frac{N_{t+1}}{N_t \theta_1 \theta_2} = 2.777778$$

1.7 Projecting into the future for model 2

The equation $N_{t+1} = \theta_1 \theta_2 N_{1997} * 2.777778$ describes the rate of change in population size N from year to the next year. Then, how can we predict the population size says for 40 years.

$$N_{1998} = \theta_1 \theta_2 N_{1997} * 2.777778 = 8$$

$$N_{1999} = \theta_1 \theta_2 N_{1997} * 2.777778 = 16$$

$$N_{2000} = \theta_1 \theta_2 N_{1997} * 2.777778 = 32$$

so that the general rule will be

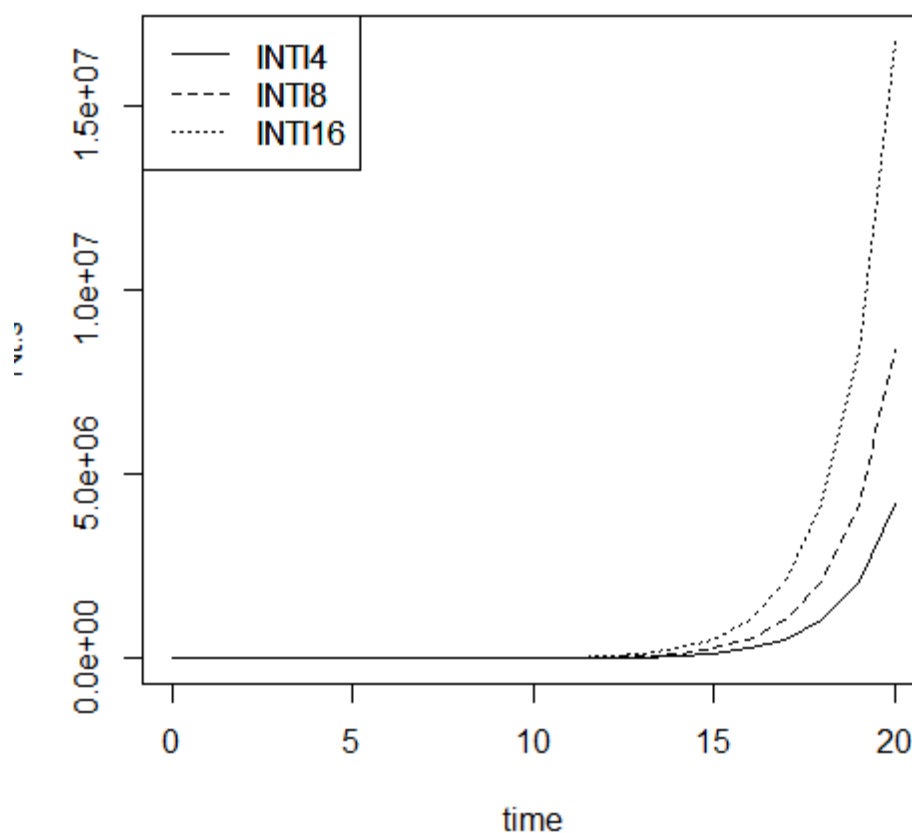
$$N_t = N_0 \theta_1^t \theta_2^t \lambda^t$$

R code to project for the model 2 for 20 years with initial three initial population size.

```
> theta<-0.8
> theta1<-0.9
> lambda<-2.7778
> N0<-c(4,8,16)
> time<-0:20
> Nt.s<-sapply(N0,function(n)n*lambda^time*theta^time*theta1^time)
> Nt.s
```

	[,1]	[,2]	[,3]
[1,]	4.000000e+00	8.000000e+00	1.600000e+01
[2,]	8.000064e+00	1.600013e+01	3.200026e+01
[3,]	1.600026e+01	3.200051e+01	6.400102e+01
[4,]	3.200077e+01	6.400154e+01	1.280031e+02
[5,]	6.400205e+01	1.280041e+02	2.560082e+02
[6,]	1.280051e+02	2.560102e+02	5.120205e+02
[7,]	2.560123e+02	5.120246e+02	1.024049e+03
[8,]	5.120287e+02	1.024057e+03	2.048115e+03
[9,]	1.024066e+03	2.048131e+03	4.096262e+03
[10,]	2.048147e+03	4.096295e+03	8.192590e+03
[11,]	4.096328e+03	8.192655e+03	1.638531e+04
[12,]	8.192721e+03	1.638544e+04	3.277088e+04
[13,]	1.638557e+04	3.277115e+04	6.554229e+04
[14,]	3.277141e+04	6.554282e+04	1.310856e+05
[15,]	6.554334e+04	1.310867e+05	2.621734e+05
[16,]	1.310877e+05	2.621755e+05	5.243509e+05
[17,]	2.621776e+05	5.243551e+05	1.048710e+06

```
[18,] 5.243593e+05 1.048719e+06 2.097437e+06
[19,] 1.048727e+06 2.097454e+06 4.194908e+06
[20,] 2.097471e+06 4.194942e+06 8.389883e+06
[21,] 4.194975e+06 8.389950e+06 1.677990e+07
> matplot(time, Nt.s, type = "l", ylab = "Nt.s", col = 1)
legend("topleft", legend=c("INTI4","INTI8","INTI16"),lty=1:3)
```



This suggests that the inclusion of parameters indicating the probability of survival (climatic changes) and degradation as a result of mans activities has not done any harm in modeling.

1.7 Motivating Continuous Exponential Growth for Model 2

Exponential growth is proportional instantaneous change over an instant. If we assume that population divides instantaneously, then many cells are dividing each fraction of a seconds. This would means that we have infinitely large number of time steps between $t=0$, and $t=1$ day. If you think a population increases at an annual growth rate $\lambda=2$. (i.e. 100% increment).

$$N_t = N_0 \theta_1 \theta_2 \lambda$$

Where $\lambda = 1 + rd$

So that rd is discrete growth increment

$rd = 1$

You could then do growth over two time steps. And

λ^2

$$N_t = N_0 \theta_1 \theta_2 \left(1 + \frac{1}{2}\right)^2$$

What will happen if we keep increasing the number of time step

$$N_t = N_0 \theta_1 \theta_2 \left(1 + \frac{rd}{n}\right)^n$$

The question is, what will happen to $\left(1 + \frac{rd}{n}\right)^n$ as n goes to infinite $\lim_{n \rightarrow \infty} \left(1 + \frac{rd}{n}\right)^n$

This above has a simple mathematical expression and it is called exponential (e). Recall that e is one of the magic numbers in mathematics that keeps popping up everywhere. In this context, we find that large values of n

$$\lim_{n \rightarrow \infty} \left(1 + \frac{rd}{n}\right)^n = e^r$$

Where r intrinsic rate of increase

Where $r = \ln \lambda$

When we say the population is growing exponentially we represent it with

$$N_t = N_0 \theta_1^t \theta_2^t e^{rt}$$

1.8 Deriving the time derivatives for model 2

We can differentiate $N_t = N_0 \theta_1^t \theta_2^t e^{rt}$ with respect to time to get the differential equation for instantaneous population growth rate, by using chain rule, the derivatives of a product of two is the sum of the product of the derivative of one times the other original term.

$$N_t = N_0 \theta_1^t \theta_2^t e^{rt}$$

$$\frac{dN}{dt} = Vdu + Udv$$

$$u = \theta_1^t \theta_2^t$$

$$v = e^{rt}$$

$$\frac{dv}{dt} = re^{rt}$$

$u = \theta_1^t \theta_2^t$ This is also a product of two variables

$$\frac{du}{dt} = adb + bda$$

$$a = \theta_1^t$$

$$b = \theta_2^t$$

$$\frac{da}{dt} = \theta_1^t \ln \theta_1$$

$$\frac{db}{dt} = \theta_2^t \ln \theta_2$$

$$\frac{du}{dt} = \theta_1^t \theta_2^t \ln \theta_2 + \theta_1^t \theta_2^t \ln \theta_1$$

Therefore

This is the time derivative or the differential equation for model 2

1.9 Doubling Time for model 2

The doubling time of a population is the time required for a population to double in size. By double time we mean that $N_t = 2N_0$

$$N_t = N_0 \theta_1^t \theta_2^t e^{rt}$$

The time it will take the population to double in size

$$2N_0 = N_0 \theta_1^t \theta_2^t e^{rt}$$

$$2 = \theta_1^t \theta_2^t e^{rt}$$

Take natural logarithm of both sides

$$\ln 2 = t \ln \theta_1 + t \ln \theta_2 + tr$$

$$\ln 2 = t(\ln \theta_1 + \ln \theta_2 + r)$$

Divide both sides by $t(\ln \theta_1 + \ln \theta_2 + r)$

$$t = \frac{\ln 2}{t(\ln \theta_1 + \ln \theta_2 + r)}$$

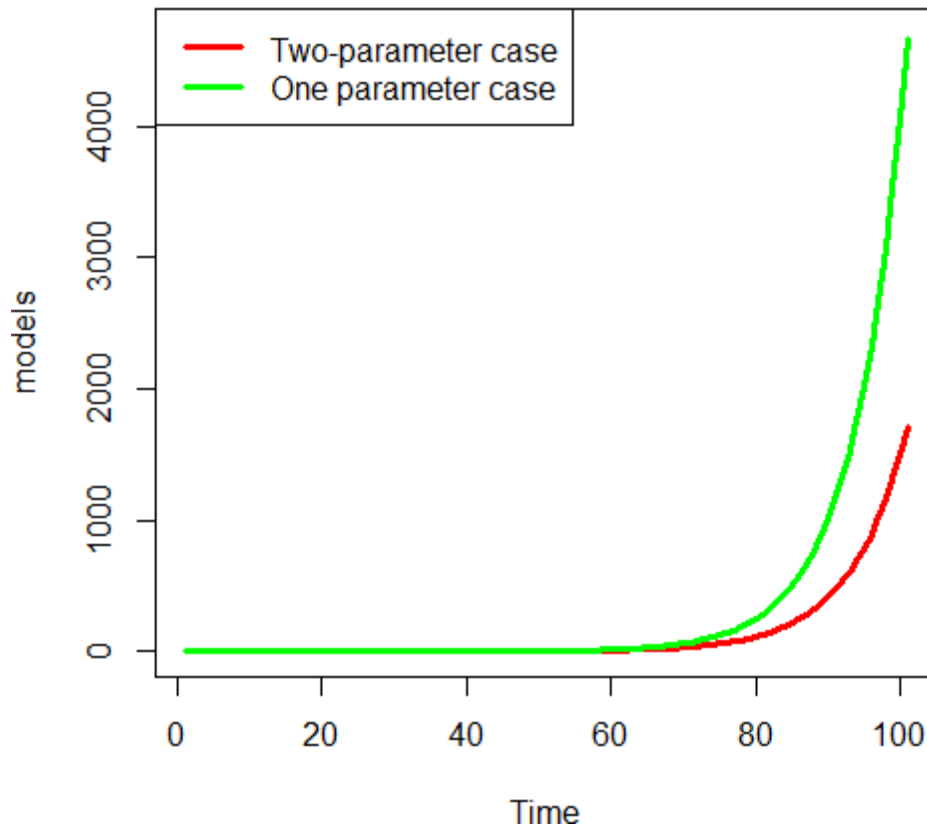
2.0 Simulating Ecology population

Let assume values for our parameters for both models and simulate ecology population size for one hundred years with R

> time<-0:100

```
> theta<-.9
> lambda<-1.27
> N0<-4
> Nt.s<-N0*theta^time*lambda^time
> time<-0:100
> theta1<-0.99
> theta<-0.9
> lambda<-1.27
> M0<-4
> Mt.s<-M0*theta1^time*theta^time*lambda^time
> danres=cbind(Nt.s,Mt.s)
> attach(danres)
> plot(Mt.s/1000,type='n',col='red',ylim=c(1,4700),
+ main='Model Comparison',xlab='Time',ylab='models')
> lines(Mt.s/1000,type='l',col='red',lwd=3)
> lines(Nt.s/1000,type='l',col='green',lwd=3)
> legend('topleft',
+ legend=c('Two-parameter case','One parameter case'),col=c('red','green'),lty=1,lwd=3)
```

Model Comparison



2.1 Comparison for both Models

The two cases are compared and they both indicate increase in population. This suggests that the inclusion of parameters indicating the probability of survival and degradation as a result of mans activities has no done any harm in modelling.

2.2 Summary / Conclusion

The two cases were compared and they both indicated an increase in population, this suggests that the inclusion of parameters indicating the probability of survival and degradation is as a result of mans activities. Therefore, learning more about population projection becomes paramount. The two cases have been compared and they suggested an increase in population over time.

In the first model we incorporated a new parameter θ_1 in growth model and θ_1 is changes due to climatic conditions, such as droughts, ocean acidification, the loss of sea ice and an increase in storms and extreme weather events that can threaten species survival. In the second model we incorporated two new parameters θ_1 and θ_2 in growth model and the parameter θ_1 is due to climatic change, the

change in climate has effect on the population size of the species and θ_2 is probability that degradation (man's activities) of the habitat will have large effect on the population size. A wide variety of animals have been hunted, or fished, beyond sustainable levels and now face possible extinction and θ_1, θ_2 are probability within $0 < \theta_1 < 1, 0 < \theta_2 < 1$. We also derive the time derivative and time for the population to double in size for both models and if θ_1, θ_2 equal one it returns us to time derivative and doubling time for exponential growth model.

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