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A Study on the Role of Fingero Imbibition Phenomenon in Displacement Process

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ABSTRACT

If a porous medium loaded up with some liquid is carried into contact with another liquid which specially wets the medium at that point there is an unconstrained stream of the wetting liquid into the medium and a counter stream of the occupant liquid from the medium. This phenomenon is called imbibitions. Thus, when a liquid contained into a porous medium is dislodged by another liquid of lesser consistency, rather than ordinary uprooting of the entire front, fingers may happen which shoot through the porous medium at moderately extraordinary speed.

This phenomenon is called fingering that have talked about an issue of the concurrent event of fingering and imbibitions in fake renewal of groundwater through split porous is the principal specialist who has assigned this compound phenomenon "Fingero-Imbibitions" and dislodging issues through porous media have increased extensive intrigue and numerous scientists have examined them from various perspectives. The current paper highlights the role of Fingero Imbibition phenomenon in Displacement process.

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KEYWORDS:

Fingero, Imbibition, Displacement

INTRODUCTION

We consider here that a cylindrical piece of homogenous porous matrix of length L filled with native liquid n, is completely surrounded by an impermeable surface except for one end of the cylinder which is designated as the imbibitions face (x=0) and this end is exposed to an adjacent formulation of the injected liquid i. If the injected liquid i is less viscous and in preferentially wetting phase, the phenomenon of fingering will occur simultaneously with imbibitions and then this arrangement describes a one-dimensional fingero-imbibitions phenomenon in which the injection is started by imbibitions and the resulting displacement produces instabilities.

Our particular interest in the present chapter is to derive a confluent hyper geometric solution for the fingero-imbibitions phenomenon in double-phase flow through unsaturated porous medium with capillary pressure. For the mathematical formulation, we assume that the Darcy's law is valid for the investigated flow system. As it was shown by Scheidegger by the flow of motion with the introduction of the fictitious relative permeability has become identical with the Buckley-Leverett description of two immiscible liquids flow through porous medium. The saturation of the wetting phase is then obtained in terms of confluent hyper geometric functions.

Assuming the validity of Darcy's law in the flow of two immiscible liquids, the seepage

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velocities of liquids n and i are expressed as

$$v_i = -\frac{k_i}{\delta_i} K \frac{\partial p_i}{\partial x} \quad (1)$$

$$v_n = -\frac{k_n}{\delta_n} K \frac{\partial p_n}{\partial x} \quad (2)$$

Where and are relative permeability's, and are the viscosities and are the weights of the infused liquid and local liquid individually; K is the porousness of the medium and x is the direct co-ordinate.

The condition of linear countercurrent imbibitions is

$$v_i = - v_n \quad (3)$$

The equation of the continuity for the injected liquid n can be written as

$$\emptyset \frac{\partial s_i}{\partial t} + \frac{\partial v_i}{\partial x} = 0 \quad (4)$$

Where is the immersion of the infused liquid I, is the porosity of the medium and t is the time. The narrow weight is characterized as the weight brokenness between the streaming stages and might be composed as

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$$p_c = p_n - p_i \quad (5)$$

We assume for definiteness that

$$p_n = -\beta g(s_i) \quad (6)$$

Where β is the positive steady narrow weight coefficient and negative sign demonstrates the course of. Substituting the estimations of from equation (1) and (2) into the equation (3),

We get

$$\frac{k_i}{\delta_i} K \frac{\partial p_i}{\partial x} + \frac{k_n}{\delta_n} K \frac{\partial p_n}{\partial x} = 0 \quad (7)$$

Eliminating between equations (5) and (7), we get

$$\left(\frac{k_i}{\delta_i} + \frac{k_n}{\delta_n}\right) K \frac{\partial p_i}{\partial x} + \frac{k_n}{\delta_n} K \frac{\partial p_c}{\partial x} = 0 \quad (8)$$

Eliminating from (1) and (8), we obtain

$$v_i = \frac{k_i k_n}{k_i \delta_n + k_n \delta_i} K \frac{\partial p_c}{\partial x} \quad (9)$$

Combining equations (4) and (9), we obtain

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$$\emptyset \frac{\partial s_i}{\partial t} + \frac{\partial}{\partial x}\left[\frac{k_i k_n}{k_i \delta_n + k_n \delta_i} K \frac{\partial p_c}{\partial x}\right] = 0 \quad (10)$$

Now substituting the v a l u e of from equation (6) into equation (10), we get

$$\emptyset \frac{\partial s_i}{\partial t} - \beta K \frac{\partial}{\partial x}\left[D(s_i).\frac{\partial s_i}{\partial x} \right] = 0 \quad (11)$$

$$\text{where } D(s_i) = \frac{k_i k_n}{k_i \delta_n + k_n \delta_i} K . g'(s_i) \quad (12)$$

The appropriate set of boundary conditions related to the phenomenon is:

$$s_i\,(0, t) = s_{io}\ ;\ (t > 0) \quad (13)$$

$$s_i\,(0, L) = s_{iL}\ ;\ (t > 0) \quad (14)$$

Where the boundary condition (13) characterizes the immersion at the imbibitions confront x = 0 and the boundary condition (14) gives the immersion at x = L, of the porous matrix.

Using the dimensionless variables

$$X = \frac{x}{L} \quad (15)$$

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$$T = \frac{\beta kt}{\emptyset L^2}$$

Where 0 and 0 < T 1, in equation (15), we obtain

$$\frac{\partial s_i}{\partial T} = \frac{\partial}{\partial X}\left[D(s_i)\ \frac{\partial s_i}{\partial X}\right] \quad (16)$$

This is a non-direct fractional differential equation governing the fingero-imbibitions phenomenon in the twofold stage course through the unsaturated permeable medium.

The boundary conditions (15) and (16) are transformed into

$$s_i\,(0,T) = s_{io} \quad (17)$$

$$s_i\,(1,T) = s_{iL} \quad (18)$$

The genuine stream of liquid in the permeable medium is rotational, yet because of homogeneity of the medium, the stream is viewed as one-dimensional and it worries with the physical significance of Laplace which happens in the equation of movement for immersion. Presently D (is a function of immersion and 0. Also in the one-dimensional stream investigation, if this sort of non-uniformity of the genuine stream isn't excessively extensive, important outcomes are gotten. Subsequently, at the stage, for definiteness, we accept that the normal estimation of D (= is consistent.

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Therefore,

$$\frac{\partial s_i}{\partial T} = \overline{D}\ \frac{\partial^2 s_i}{\partial X^2} \quad (19)$$

Together with boundary conditions (18) and (19) speaks to the boundary esteem issue. The equation (20) is the parabolic differential equation describing the immersion of the infused liquid. We infer the established solution of (20) as far as confluent hyper geometric functions.

ROLE OF FINGERO IMBIBITION PHENOMENON IN DISPLACEMENT PROCESS

Consider the accompanying limit esteem issue related with the immersion of the infused liquid:

$$\overline{D}\ \frac{\partial^2 s_i}{\partial x^2} = \frac{\partial s_i}{\partial t} \quad (21)$$

The technique is started with the gathering G, a class of transformations of one-parameter „a‟ of the form:

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$$G:\begin{cases}\bar{x} = \mathbb{C}^x(a)\,x + K^x\,(a)\\ \bar{t} = \mathbb{C}^t(a)\,t + K^t\,(a)\\ \bar{s_l} = \mathbb{C}^{s_i}(a)\,s_i + K^{s_i}\,(a)\end{cases} \quad (22)$$

Where C" s and K's are genuine esteemed differentiable functions in the genuine parameter "a".

$$\frac{\partial \bar{x}}{\partial x} = \mathbb{C}^x(a);\ \frac{\partial \bar{t}}{\partial t} = \mathbb{C}^t(a);\ \frac{\partial \bar{s_l}}{\partial u} = \mathbb{C}^{s_i}(a)$$

Now,

$$\frac{\partial \bar{s_l}}{\partial \bar{t}} = \frac{\partial \bar{s_l}}{\partial s_i} \cdot \frac{\partial s_i}{\partial \bar{t}} = \mathbb{C}^{s_i}(a)\,\frac{\partial s_i}{\partial t}\,\frac{\partial t}{\partial \bar{t}} = \frac{\mathbb{C}^{s_i}(a)}{\mathbb{C}^t(a)}\,\frac{\partial s_i}{\partial t}$$

$$\therefore \frac{\partial s_i}{\partial t} = \frac{\mathbb{C}^t(a)}{\mathbb{C}^{s_i}(a)}\,\frac{\partial \bar{s_l}}{\partial \bar{t}}$$

$$\frac{\partial \bar{s_l}}{\partial \bar{x}} = \frac{\partial \bar{s_l}}{\partial x} \cdot \frac{\partial x}{\partial \bar{x}} = \frac{\partial \bar{s_l}}{\partial s_i}\,\frac{\partial s_i}{\partial x}\,\frac{1}{\mathbb{C}^x(a)} = \frac{\mathbb{C}^{s_i}(a)}{\mathbb{C}^x(a)}\,\frac{\partial s_i}{\partial x}$$

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$$\therefore \frac{\partial s_i}{\partial x} = \frac{\mathbb{C}^x(a)}{\mathbb{C}^{s_i}(a)} \frac{\partial \overline{s_\iota}}{\partial \bar{x}}$$

$$\frac{\partial^2 s_i}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \overline{s_\iota}}{\partial \bar{x}}\right)\frac{\mathbb{C}^x(a)}{\mathbb{C}^{s_i}(a)}$$

$$= \frac{\mathbb{C}^x(a)}{\mathbb{C}^{s_i}(a)} \frac{\partial}{\partial \bar{x}}\left(\frac{\partial \overline{s_\iota}}{\partial \bar{x}}\right)\frac{\partial \bar{x}}{\partial x}$$

$$= \frac{\mathbb{C}^x(a)}{\mathbb{C}^{s_i}(a)} \frac{\partial^2 \overline{s_\iota}}{\partial \bar{x}^2}\mathbb{C}^x(a) = \frac{[\mathbb{C}^x(a)]^2}{\mathbb{C}^{s_i}(a)} \frac{\partial^2 \overline{s_\iota}}{\partial \bar{x}^2} \quad (23)$$

Substituting (3) in equation (4), yields

$$\frac{[\mathbb{C}^x(a)]^2}{\mathbb{C}^{s_i}(a)} \bar{D} \frac{\partial^2 \overline{s_\iota}}{\partial \bar{x}^2} = \frac{\mathbb{C}^t(a)}{\mathbb{C}^{s_i}(a)} \frac{\partial \overline{s_\iota}}{\partial \bar{t}} \quad (24)$$

Equation (24) is said to be invariantly changed at whatever point,

$$\frac{[\mathbb{C}^x(a)]^2}{\mathbb{C}^{s_i}(a)} = 1 \text{ and } \frac{\mathbb{C}^t(a)}{\mathbb{C}^{s_i}(a)} = 1$$

$$[\mathbb{C}^x(a)]^2 = \mathbb{C}^{s_i}(a) \text{ and } \mathbb{C}^t(a) = \mathbb{C}^{s_i}(a)$$

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$$\therefore \quad [\mathbb{C}^{x}(a)]^2 = \mathbb{C}^{t}(a) = \mathbb{C}^{s_i}(a)$$

$$\therefore \quad \mathbb{C}^{x}(a) = \sqrt{\mathbb{C}^{s_i}(a)}$$

From (25), we get

$$\overline{D}\ \frac{\partial^2 \overline{s_i}}{\partial \bar{x}^2} = \frac{\partial \overline{s_i}}{\partial \bar{t}} \qquad (25)$$

The invariance of (25) implies

$$\bar{x} = \sqrt{\mathbb{C}^{s_i}(a)}\ x + K^{x}(a)$$

$$\bar{t} = \mathbb{C}^{s_i}(a)\ t + K^{s_i}(a)$$

$$\overline{s_i} = \mathbb{C}^{s_i}(a)\ s_i + K^{s_i}(a)$$

$$\overline{s_i} = \mathbb{C}^{s_i}(a)\ s_i + K^{s_i}(a) \qquad \textbf{(26)}$$

1. We have $s_i(0, t) = s_{i0}$

$$\text{At } x = 0 \implies \bar{x} = 0$$

Now $\overline{s_i}(\bar{x}, \bar{t}) = \mathbb{C}^{s_i}(a)\ s_i(x, t) + K^{s_i}(a)$

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$$\bar{s_i}(0, \bar{t}) = \mathbb{C}^{s_i}(a)\, s_i(0, t) + K^{s_i}(a)$$

$$= \mathbb{C}^{s_i}(a) s_{i0} + K^{s_i}(a) \quad \textbf{(27)}$$

2. We have $s_i(L, t) = s_{iL}$

$$\text{At } x = L \implies \bar{x} = \sqrt{\mathbb{C}^{s_i}(a)}\, L + K^L(a)$$

Now, $\bar{s_i}(\bar{x}, \bar{t}) = \mathbb{C}^{s_i}(a)\, s_i(x, t) + K^{s_i}(a)$

$$\bar{s_i}(\sqrt{\mathbb{C}^{s_i}(a)}\, L + K^L(a), \bar{t}) = \mathbb{C}^{s_i}(a)\, s_i(L, t) + K^{s_i}(a)$$

$$\bar{s_i}(\sqrt{\mathbb{C}^{s_i}(a)}\, L + K^L(a), \bar{t}) = \mathbb{C}^{s_i}(a)\, s_i(L, t) + K^{s_i}(a)$$

$$= \mathbb{C}^{s_i}(a) s_{iL} + K^{s_i}(a) \quad \textbf{(28)}$$

The boundary conditions (27) and (28) are invariantly valid if

$$K^x(a) = K^{s_i}(a) = K^t(a) = 0 \quad \textbf{(28)}$$

In this way, we have the accompanying limit esteemed issue (B.V.P)

$$\bar{D}\, \frac{\partial^2 \bar{s_i}}{\partial \bar{x}^2} = \frac{\partial \bar{s_i}}{\partial \bar{t}}$$

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Where $\bar{s}_i(0, \bar{t}) = \mathbb{C}^{s_i}(a) s_{i0}$

$$\bar{s}_i(L, \bar{t}) = \mathbb{C}^{s_i}(a) s_{iL} \quad \textbf{(29)}$$

We have,

$$s_i\left(\sqrt{\mathbb{C}^{s_i}(a)}\, x,\ \mathbb{C}^{s_i}(a) t\right) = \mathbb{C}^{s_i}(a)\, s_i(x, t)$$

$$\frac{\partial}{\partial a} s_i\left(\sqrt{\mathbb{C}^{s_i}(a)}\ x, \mathbb{C}^{s_i}(a) t\right) = \frac{\partial}{\partial a} \mathbb{C}^{s_i}(a)\, s_i(x, t)$$

$$\frac{\partial}{\partial x} s_i\left(\sqrt{\mathbb{C}^{s_i}(a)}\, x, \mathbb{C}^{s_i}(a) t\right) \frac{\partial x}{\partial a} + \frac{\partial}{\partial t} s_i\left(\sqrt{\mathbb{C}^{s_i}(a)}\ x, \mathbb{C}^{s_i}(a) t\right) \frac{\partial t}{\partial a} = s_i(x, t) \quad \textbf{(30)}$$

Now,

$$\bar{x} = \sqrt{\mathbb{C}^{s_i}(a)}\ x \Rightarrow x = \frac{\bar{x}}{\sqrt{\mathbb{C}^{s_i}(a)}} \text{ and } \bar{t} = \mathbb{C}^{s_i}(a)\, t \Rightarrow t\ \frac{\bar{t}}{\mathbb{C}^{s_i}(a) t}$$

$$\frac{\partial x}{\partial a} = \bar{x} \cdot \frac{-1}{2} \left[\mathbb{C}^{s_i}(a)\right]^{-3/2} \text{ and } \frac{\partial t}{\partial a} = \frac{-\bar{t}}{[\mathbb{C}^{s_i}(a)]^2} \quad \textbf{(31)}$$

Substituting (30) in (31), we get

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$$\frac{\partial}{\partial x} s_i(\bar{x}, \bar{t})\, \bar{x}\, \frac{-1}{2}\left[\mathbb{C}^{s_i}(a)\right]^{-3/2} + \frac{\partial}{\partial t} s_i(\bar{x}, \bar{t}) \frac{-\bar{t}}{[\mathbb{C}^{s_i}(a)]^2} = s_i(x, t)$$

Replacing by x and by t, we get

$$\frac{-1}{2}\left[\mathbb{C}^{s_i}(a)\right]^{-3/2} x \frac{\partial}{\partial x} s_i(x, t) - \frac{\bar{t}}{[\mathbb{C}^{s_i}(a)]^2} \frac{\partial}{\partial t} s_i(x, t) = s_i(x, t)$$

$$\text{As } \mathbb{C}^{s_i}(a) \longrightarrow 1$$

$$\frac{x}{2} s_i(x, t) + t \frac{\partial}{\partial t} s_i(x, t) = -s_i(x, t) \quad \textbf{(.32)}$$

The characteristics equations are

$$\frac{dx}{x/2} = \frac{dt}{t} = \frac{ds_i}{-1}$$

$$2\int \frac{dx}{x} = \int \frac{dt}{t} \qquad -\int \frac{dt}{t} = \int ds_i$$

$$2\log x = \log t + \log z^2 \qquad \log f(z) - \log t = s_i$$

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$$\log x^2 = \log t\, z^2 \qquad \frac{f(z)}{t} = e^{s_i}$$

$$x^2 = t\, z^2 \qquad f(z) = e^{s_i}\, t$$

$$z = \frac{x}{\sqrt{t}}$$

The General solution has the form

$$s_i(x,\ t) = \frac{1}{\sqrt{t}} f(z) \text{ where } z = \frac{x}{\sqrt{t}} \quad \textbf{(33)}$$

CONCLUSION

The numerical formulation of the phenomenon of the Fingero-Imbibitions yields a non-straight incomplete differential equation which is transformed into a conventional differential equation by utilizing a one parameter bunch hypothesis. This later equation is decreased into the confluent hyper geometric differential equation with suitable boundary conditions by utilizing some standard transformations. An established solution as far as confluent hyper geometric functions, of the later has been acquired. This established solution speaks to the immersion of the wetting stage at whatever point the immersion coefficient is certain consistent.

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