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ROUGH $2-(0-or\ 1-)$ PRIME IDEALS AND $m_2-(m_0-\ or\ m_1-)$ SYSTEMS OF SEMIRINGS

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Abstract

This paper presents the concept of $\mathbf{2}-(\mathbf{0}-\boldsymbol{or}\ \mathbf{1}-)$ prime ideals in semiring and $\boldsymbol{m_2}-(\boldsymbol{m_0}-\ \boldsymbol{or}\ \boldsymbol{m_1}-)$ system in semirings through rough set theory. The result that is "every rough $\mathbf{0}$-prime ideal is also a rough $\mathbf{2}$-prime ideal and (rough $\mathbf{1}$-prime ideal) in semiring" is also proved . Finally to show that an ideal P is $\mathbf{2}-(\mathbf{0}-\boldsymbol{or}\ \mathbf{1}-)$ prime ideal if and if the lower and upper approximation of $\boldsymbol{R\backslash P}$ and upper approximation of $\boldsymbol{R\backslash P}$ are $\boldsymbol{m_2}-(\boldsymbol{m_0}-\ \boldsymbol{or}\ \boldsymbol{m_1}-)$ systems in semiring R.

Keywords: Semirings; congruence relation; rough $2-(0-or\ 1-)$ prime ideals; rough $m_2-(m_0-\ or\ m_1-)$ system

1 INTRODUCTION

The concept of semiring was introduced by H.S.Vandiver[17]. Iseki[6] introduced prime ideals in semirings.Gupta and Chaudari[5] have developed some properties of prime ideals in semirings. Dheena and Coumaressane[4] introduced fuzzy 2-(0-or1-) prime ideals in semirings.
Pawlak [11-13] launched rough set theory as a frame work for the construction of approximations of concepts when only incomplete information is available. Many others studied the algebraic properties of rough set. So many authors presented the articles in algebraic approach of rough sets in different algebraic structures such as [1,2,3,7,8,9,10,13,15]. Thillaigovindan and Subha[14], introduced rough ideals in near-rings

In this paper we introduce the notion of rough prime $2-(0-or\ 1-)$ ideals and rough $m_2-(m_0-\ or\ m_1-)$ systems in semirings. We study some of the semiring theory through rough set concepts. We have shown that every rough 0-prime ideal is also a rough 2-prime ideal (rough 1-prime ideal) in semirings. But the converse is not true in general, which is shown by an example. We find the condition under which a rough 2-(1−)prime ideal to be a rough 0-prime ideal of semirings. We have also shown that $\theta(P)$ is rough 2-prime ideal (1-prime ideal, 0-prime ideal) of R if and only if $R\backslash\theta(P)=(R\backslash\underline{\theta}(P),R\backslash\underline{\theta}(P))$ is a rough m_2-system (m_1-system, m_0 system) of R. Finally we have shown that when $\theta(Q)$ and $\theta(M)$ are a rough ideal and a rough m_2-system (m_1-system) of R respectively, such that $\hat{\theta}(Q)\cap\hat{\theta}(M)\neq\emptyset$,then there exists a rough 2-prime ideal (1- prime ideal) $\theta(P)$ of R such that $\theta(Q)\subseteq\theta(P)$ with $\theta(P)\cap\theta(M)=\emptyset$.

2 PRELIMINARIES

In this section some definitions and results are reproduced, which are proposed by pioneers in this field earlier and are necessary for the development of the new results.

A *semiring* is a system consisting of a nonempty set *R* together with binary operations on *R* called *addition* and *multiplication* such that $(R,+)$ is a semigroup; $(R,.)$ is a semigroup and $a.(b+c)=a.b+a.c$ and $(b+c).a=b.a+c.a$ for all $a,b,c\in R$. A semiring *R* may have an *identity 1*,defined by $1.a=a=a.1$ and a zero 0, defined by $0+a=a=a+0$ and $a.0=0=0.a$ for all $a\in R$. From now on we write *R* for semirings. A nonempty subset *I* of *R* is said to be a *left*(resp. *right*) *ideal* if $x,y\in I$ and $r\in R$ imply that $x+y\in I$ and $r.x\in I$ and (resp. $x.r\in I$). *I* is

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Volume 02 Issue 11
November-2014

said to be *two –sided ideal* or simply *ideal* of *R*, if *I* is both left and right ideal of *R*. The ideal generated by $a \in R$ is defined as the smallest ideal of *R*, which contains a and is denoted by $< a >$. For the sake of convenience hereafter we write ab instead of $a \bullet b$.

3. CONGRUENCE RELATION AND APPROXIMATION SPACE

In this section we list out some definitions, theorems and lemmas for the sake of continuity and completeness and to be used in the latter sections.

Let U be a universal set. An *equivalence relation* θ on R is a reflexive, symmetric and transitive binary relation on U. The set of elements of U that are related to $x \in U$ is called the equivalence class of x and is denoted by $[x]_\theta$.

Definition 3.1[13] A pair (U, θ) wher $U \neq \phi$ and θ is an equivalence relation on U, is called an *approximation space*

Definition 3.2[13] For an approximation space (U, θ) by a *rough approximation* in (U, θ) we mean a mapping ρ: $\wp(U) \to \wp(U) \times \wp(U)$ defined as $\rho(X) = \left(\underline{\rho}(X), \overline{\rho}(X)\right)$ for X⊆U where $\underline{\rho}(X) = \{x \in U | [x]_\theta \subseteq U\}$, and $\overline{\rho}(X) = \{x \in U | [x]_\theta \cap U \neq \phi\}$, $\underline{\rho}(X)$ is called a *lower rough approximation* of *X* in (U, θ) where as $\overline{\rho}(X)$ is called an *upper rough approximation* of *X* in (U, θ). Hereafter we use $\underline{\theta}(X), \overline{\theta}(X)$ and $\theta(X)$ instead of $\underline{\rho}(X), \overline{\rho}(X)$ and $\rho(X)$ respectively.

Definition 3.3[13] Given an approximation space (U, θ) a pair $(A, B) \in \wp(U) \times \wp(U)$ is called a *rough set* in (U, θ) if and only if $(A, B) = \theta(X)$ for some $X \subseteq U$.

If A nd B are any two subsets of R, then $AB = \{ab | a \epsilon A, b \epsilon B\}$.

Definition 3.4 [13] Let θ be an equivalence relation on R. θ is called a *congruence relation* if $(a, b) \in \theta$ implies

(i) $(a + x, b + x) \in \theta$; (ii) $(x + a, x + b) \in \theta$; (iii) $(ax, bx) \in \theta$ and (iv) $(xa, xb) \in \theta$, for all $x \in R$.

The following theorem is an immediately consequence of Definition 2.4.

Theorem 3.5.[13] *Let θ be a congruence relation on a semiring R.* Then $(a, b), (c, d) \in \theta$ implies $(a + c, b + d) \in \theta$, $(ac, bd) \in \theta$ for all $a, b, c, d \in R$.

Lemma 3.6.[13] *Let θ be a congruence relation on R. If $a, b \in R$, then*

(i) $[a]_\theta + [b]_\theta \subseteq [a + b]_\theta$

(ii) $[a]_\theta . [b]_\theta \subseteq [ab]_\theta$

Proof. (i) Let $x \in R$. Suppose $x \in [a]_\theta + [b]_\theta$. Then there exist $y, z \in R$ such that $y \in [a]_\theta$, $z \in [b]_\theta$ and $x = y + z$. This means that $(a, y), (b, z) \in \theta$ and hence $(a + b, y + z) = (a + b, x) \in \theta$. Thus $x \in [a + b]_\theta$ and hence $[a]_\theta + [b]_\theta \subseteq [a + b]_\theta$.

(ii)Let $z = xy \in [a]_\theta . [b]_\theta$. Then $x \in [a]_\theta$ and $y \in [b]_\theta$. This implies that $(a, x) \in \theta$ and $(b, y) \in \theta$. Since θ is a congruence relation, $(ab, xy) \in \theta$.Thus $z = xy \in [ab]_\theta$ and hence $[a]_\theta . [b]_\theta \subseteq [ab]_\theta$.

A congruence relation θ on R is called complete if $[a]_\theta + [b]_\theta = [a + b]_\theta$ and $[a]_\theta . [b]_\theta = [ab]_\theta$.

Definition 3.7. Let θ be a congruence relation on R and A a subset of R. Then the sets $\underline{\theta}(A) = \{x \in R / [x]_\theta \subseteq A\}$ and $\overline{\theta}(A) = \{x \in S / [x]_\theta \cap A \neq \phi\}$ are called the lower and upper approximations of the set A, respectively. Let A be any subset of R. $\theta(A) = (\underline{\theta}(A), \overline{\theta}(A))$ is called a rough set with repect to θ if $\underline{\theta}(A) \neq \overline{\theta}(A)$.

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Lemma 3.8. *For any approximation space* (R,θ) and $P \subseteq R$, *the following hold:*
(i) $\underline{\theta}(\mathrm{R}\backslash\mathrm{P}) = \mathrm{R}\backslash\overline{\theta}(\mathrm{P})$; (ii) $\overline{\theta}(\mathrm{R}\backslash\mathrm{P}) = \mathrm{R}\backslash\underline{\theta}(\mathrm{P})$; (iii) $\overline{\theta}(\mathrm{P}) = \left(\underline{\theta}(\mathrm{P}^{\mathrm{c}})\right)^{\mathrm{c}}$; (iv) $\underline{\theta}(\mathrm{P}) = \left(\overline{\theta}(\mathrm{P}^{\mathrm{c}})\right)^{\mathrm{c}}$
Proof. The proof is obvious and hence omitted.
Theorem 3.9. [16] *Let* θ *and* ψ *be congruence relations on* R *and let* A *and* B *be nonempty subsets of* R. *Then*

(i) $\underline{\theta}(\mathrm{A}) \subseteq \mathrm{A} \subseteq \overline{\theta}(\mathrm{A})$
(ii) $\underline{\theta}(\emptyset) = \emptyset = \overline{\theta}(\emptyset)$
(iii) $\underline{\theta}(\mathrm{R}) = \mathrm{R} = \overline{\theta}(\mathrm{R})$
(iv) $\overline{\theta}(\mathrm{A} \cup \mathrm{B}) = \overline{\theta}(\mathrm{A}) \cup \overline{\theta}(\mathrm{B})$
(v) $\underline{\theta}(\mathrm{A} \cap \mathrm{B}) = \underline{\theta}(\mathrm{A}) \cap \underline{\theta}(\mathrm{B})$
(vi) $A \subseteq B$ implies $\underline{\theta}(\mathrm{A}) \subseteq \underline{\theta}(\mathrm{B})$ and $\overline{\theta}(\mathrm{A}) \subseteq \overline{\theta}(\mathrm{B})$
(vii) $\underline{\theta}(\mathrm{A} \cup \mathrm{B}) \supseteq \underline{\theta}(\mathrm{A}) \cap \underline{\theta}(\mathrm{B})$
(viii) $\overline{\theta}(\mathrm{A} \cap \mathrm{B}) \subseteq \overline{\theta}(\mathrm{A}) \cap \overline{\theta}(\mathrm{B})$
(ix) $\theta \subseteq \psi$ implies $\underline{\psi}(\mathrm{A}) \subseteq \underline{\theta}(\mathrm{A})$ and $\overline{\theta}(\mathrm{A}) \subseteq \overline{\psi}(\mathrm{A})$
(x) $\overline{(\theta \cap \psi)}(\mathrm{A}) = \overline{\theta}(\mathrm{A}) \cap \overline{\psi}(\mathrm{A})$
(xi) $\overline{(\theta \cap \psi)}(\mathrm{A}) \subseteq \underline{\theta}(\mathrm{A}) \cap \underline{\theta}(\psi)$
(xii) $\underline{\theta}\left(\underline{\theta}(\mathrm{A})\right) = \underline{\theta}(\mathrm{A})$
(xiii) $\overline{\theta}\left(\overline{\theta}(\mathrm{A})\right) = \overline{\theta}(\mathrm{A})$
(xiv) $\overline{\theta}\left(\underline{\theta}(\mathrm{A})\right) = \underline{\theta}(\mathrm{A})$
(xv) $\underline{\theta}\left(\overline{\theta}(\mathrm{A})\right) = \overline{\theta}(\mathrm{A})$.

Definition 3.10.[16] Let A be any subset of R and (R,θ) be a rough approximation space. If $\underline{\theta}(\mathrm{A})$ and $\overline{\theta}(\mathrm{A})$ are ideals , then $\underline{\theta}(\mathrm{A})$ is called a *lower and upper rough ideal* and $\overline{\theta}(\mathrm{A})$ is called an *upper rough ideal* of R, respectively. $\theta(A) = (\underline{\theta}(\mathrm{A}), \overline{\theta}(\mathrm{A}))$ is called *rough ideal* of R.

Lemma 3.11.[16] *If* I *and* J *are ideals of* R *and* $\underline{\theta}(I \cap J)$ *is a nonempty set, then* $\left(\underline{\theta}(I \cap J), \overline{\theta}(I \cap J)\right)$ *is a rough ideal of* R.

Theorem 3.12.[14] *Let* θ *be a relation and I, J be subsets of R.*

(i) *If* I *and* J *are left ideals of R, then* $\left(\theta(I):\theta(I)\right)_l$ *is a rough ideal of R.*
(ii) *If* I *and* J *are right ideals of R, then* $\left(\theta(I):\theta(I)\right)_r$ *is rough ideal of R.*

Theorem 3.13.[14] *Let* θ *be a congruence relation and I, J be subsets of R.*

(i) *If* $\overline{\theta}(I)$ *and* $\overline{\theta}(J)$ *are left-ideals of R, then* $\left(\overline{\theta}(I):\overline{\theta}(J)\right)_l$ *is an ideal of R.*
(ii) *If* $\underline{\theta}(I)$*and* $\underline{\theta}(J)$ *are left-ideals of R with* $\underline{\theta}(I)$ *and* $\underline{\theta}(J)$ *nonempty subsets of R, then* $\left(\underline{\theta}(I):\underline{\theta}(J)\right)_l$ *is an ideal of R.*

Thorem 3.14. [14] *Let* θ *be a congruence relation*

i) *If* $\overline{\theta}(I)$ *is a left k-ideals and* $\overline{\theta}(J)$ *is a left ideal of R, then* $\left(\overline{\theta}(I):\overline{\theta}(J)\right)_l$ *is a k-ideal of R.*

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ii) *If* $\underline{\theta}(I)$ *is a right k-ideal and* $\underline{\theta}(J)$ *is a right ideal , then* $\left(\underline{\theta}(I):\underline{\theta}(J)\right)_r$ *is a k-ideal of R.*

4.ROUGH $2-(0-or\ 1-)$ PRIME IDEALS AND $m_2-(m_0-\ or\ m_1-)$ SYSTEMS IN SEMIRINGS

In this section we study the concept of $2-(0-or\ 1-)$ prime ideals in semiring and $m_2-(m_0-\ or\ m_1-)$ system in semirings through rough set theory. We have shown that I is a k-ideal of a semiring R if and only if the lower and upper approximations of I with respect to congruence relation θ are k-ideals in semirings. Finally we have also shown that an ideal P is $2-(0-or\ 1-)$ prime ideal if and if the lower and upper approximation of $R\backslash P$ and upper approximation of $R\backslash P$ are $m_2-(m_0-\ or\ m_1-)$ systems in semiring R.

Throughout this section we assume that every lower approximation of any subset A is nonempty unless otherwise mentioned.

Definition 4.1. Let R be a semiring. If $A \subseteq R$, then the ideal (k-ideal) of R generated by A will be denoted $\langle A\rangle(\langle A\rangle_k)$.

If $A=\{a\}$. We write $\langle a\rangle$ instead of $\langle\{a\}\rangle$for convenience.

Definition 4.2. An ideal P of R is called a $0-(2-)$ *prime* of R, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

An ideal P of R is called a *1-prime ideal* if for any k-ideal A of R and for any ideal B of R, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

Lemma 4.3 Let θ be a congruence relation and I be a subset of R. If $\theta(I)$ is a *rough 0-prime ideal* of R then $\theta(I)$ is *a rough 2-prime (1-prime) ideal* of R.

Proof . Obvious.

Lemma 4.4. *Let* θ *be a congruence relation and* I *be a subset of* R.

(i) *If* $\overline{\theta}(I)$ *is a 0-prime ideal of R, then* $\overline{\theta}(I)$ *is a 2-prime ideal (1- prime ideal)of R.*
(ii) *If* $\underline{\theta}(I)$ *is a 0-prime ideal of R,with* $\underline{\theta}(I)$ *nonempty, then* $\underline{\theta}(I)$ *is a 2-prime ideal (1-prime ideal) of R.*

Proof. It is obvious.

The reverse parts of Lemma 4.4 are not true in general which is shown in the following example.

Example 4.5. Consider the semiring $R=\{0,1,2,3\}$, where " +" and "." are defined as follows.

+	0	1	2	3

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November-2014

0	0	1	2	3
1	1	2	3	3
2	2	3	3	3
3	3	3	3	3

.	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	3
3	0	3	3	3

Define a congruence relation θ in R as $(x, y) \in \theta \Leftrightarrow x + A = y + A$, where A is an ideal of R. One can check by routine calculation that $I = \{0,3\}$ and $J = \{0,2,3\}$ are ideals in R. Now $\overline{\theta}(I) = \{0,3\}$, $\overline{\theta}(J) = \{0,2,3\}$. Then $\overline{\theta}(J).\overline{\theta}(J) \subseteq \overline{\theta}(I)$ and $\overline{\theta}(J) \nsubseteq \overline{\theta}(I)$. thus $\overline{\theta}(I)$ is not 0-prime. But $\overline{\theta}(I)$is a 2-prime ideal of R. Thus $\theta(I)$is a rough 2-prime ideal of R but not a rough ideal of R.

Theorem 4.6 *Let θ be a congruence relation and P be any k-ideal of R. $\theta(P)$ is a rough 0-prime ideal if and only if $\theta(P)$ is a rough 2-prime ideal of R.*

Proof. Assume that $\theta(P)$ is a rough k-ideal of R which is also a rough 2-prime ideal. Suppose $\overline{\theta}(I)$ and $\overline{\theta}(J)$ are ideals of R such that $\overline{\theta}(I).\overline{\theta}(J) \subseteq \overline{\theta}(P)$.
Now $\overline{\theta}(I).\overline{\theta}(J) \subseteq \overline{\theta}(P)$ implies $\overline{\theta}(I) \subseteq \left(\overline{\theta}(P):\overline{\theta}(J)\right)_l$. By Theorem 3.13 $\left(\overline{\theta}(P):\overline{\theta}(J)\right)_l$ is a k-ideal of R. This means $\langle\overline{\theta}(I)\rangle_k \subseteq \left(\overline{\theta}(P):\overline{\theta}(J)\right)_l$. Hence $\langle\overline{\theta}(I)\rangle_k\overline{\theta}(J) \subseteq \overline{\theta}(P)$. Now $\langle\overline{\theta}(I)\rangle_k\overline{\theta}(J) \subseteq \overline{\theta}(P)$ implies $\overline{\theta}(J) \subseteq \left(\overline{\theta}(P):\langle\overline{\theta}(I)_k\rangle\right)_r$. By Theorem3.5.19 $\langle\overline{\theta}(J)\rangle_k \subseteq \left(\overline{\theta}(P):\langle\overline{\theta}(I)_k\rangle\right)_r$. This means that $\langle\overline{\theta}(I)\rangle_k.\langle\overline{\theta}(J)\rangle_k \subseteq \overline{\theta}(P).\overline{\theta}(P)$ being k-ideal, $\langle\overline{\theta}(I)\rangle_k \subseteq \overline{\theta}(P)$ or $\langle\overline{\theta}(J)\rangle_k \subseteq \overline{\theta}(P)$.
Thus $\overline{\theta}(I) \subseteq \langle\overline{\theta}(I)\rangle_k \subseteq \overline{\theta}(P)$ or $\overline{\theta}(J) \subseteq \langle\overline{\theta}(J)\rangle_k \subseteq \overline{\theta}(P)$. Hence $\overline{\theta}(P)$ is a 0-prime ideal of R. Similarly it can be proved that $\underline{\theta}(P)$ is a 0-prime ideal of R. This shows that $\theta(P)$ is a rough 0-prime ideal of R.

Theorem 4.7. *Let θ be a congruence relation and let P any subset of R,*

(i) *Let $\overline{\theta}(P)$ be a k-ideal of R. $\overline{\theta}(P)$ is a 0-prime ideal if and only if $\overline{\theta}(P)$ is a 2-prime ideal.*
(ii) *Let $\underline{\theta}(P)$ be a k-ideal with $\underline{\theta}(P)$ is nonempty , $\underline{\theta}(P)$ is a 0-prime ideal if and only if $\underline{\theta}(P)$ is a 2-prime ideal.*

Proof. Assume that $\overline{\theta}(P)$is a 2-prime ideal and $\overline{\theta}(P)$ is a k-ideal of R. Suppose $\overline{\theta}(I)$ and $\overline{\theta}(J)$ are ideals of R such that $\overline{\theta}(I).\overline{\theta}(J) \subseteq \overline{\theta}(P)$. Now $\overline{\theta}(I).\overline{\theta}(J) \subseteq \overline{\theta}(P)$ implies $\overline{\theta}(I) \subseteq \left(\overline{\theta}(P):\overline{\theta}(J)\right)_l$. By Theorem 3.5.12 $\left(\overline{\theta}(P):\overline{\theta}(J)\right)_l$ is a k-ideal of R. This means $\langle\overline{\theta}(I)\rangle_k \subseteq \left(\overline{\theta}(P):\overline{\theta}(J)\right)_l$. Hence $\langle\overline{\theta}(I)\rangle_k\overline{\theta}(J) \subseteq \overline{\theta}(P)$. Now $\langle\overline{\theta}(I)\rangle_k\overline{\theta}(J) \subseteq \overline{\theta}(P)$, we have $\overline{\theta}(J) \subseteq \left(\overline{\theta}(P):\langle\overline{\theta}(I)_k\rangle\right)_r$. By Theorem3.5.10 $\langle\overline{\theta}(J)\rangle_k \subseteq \left(\overline{\theta}(P):\langle\overline{\theta}(I)_k\rangle\right)_r$. This means that $\langle\overline{\theta}(I)\rangle_k.\langle\overline{\theta}(J)\rangle_k \subseteq \overline{\theta}(P).\overline{\theta}(P)$ being k-ideal, $\langle\overline{\theta}(I)\rangle_k \subseteq \overline{\theta}(P)$ or $\langle\overline{\theta}(J)\rangle_k \subseteq \overline{\theta}(P)$. Thus $\overline{\theta}(I) \subseteq \langle\overline{\theta}(I)\rangle_k \subseteq \overline{\theta}(P)$ or $\overline{\theta}(J) \subseteq \langle\overline{\theta}(J)\rangle_k \subseteq \overline{\theta}(P)$. Hence $\overline{\theta}(P)$ is a 0-prime ideal of R.

Definition 4.8. $M \subseteq R$ is called an m_0-system if for every $a, b \in M$, there exists $x \in R$ such that $axb \in M$.

$M \subseteq R$ is called an m_1-system if for every $a, b \in M$ there exists $a_1 \in \langle a\rangle_k$ and there exists $b_1 \in \langle b\rangle$ such that $a_1b_1 \in M$.

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$M \subseteq R$ is called an m_2-system if for every $a, b \in M$ there exists $a_1 \in \langle a \rangle_k$ and there exists $b_1 \in \langle b \rangle_k$ such that $a_1 b_1 \in M$.

Lemma 4.9 Every m_0 −system is an m_2 −system(m_1 − system).

Proof. Assume that M is an m_0 −system. Let $a, b \in M$. Since M is an m_0 −system, there exists $x \in R$ such that $axb \in M$.

If $a_1 = ax$ and $b_1 = b$ then $a_1 b_1 \in M$, where $a_1 = ax \in \langle a \rangle_k$ and $b_1 = b \in \langle b \rangle_k$. Thus M is an m_2 − system.

But the converse of the above Lemma need not be true as the following.

Example 4.10. Consider the semiring R as in Example 3.5. Clearly $M = \{1,2\}$ is an m_2 −system but not an m_0 −system.

Theorem 4.11. *Let θ be a congruence relation on R and let P be a subset of R. $\theta(P)$ is rough 2-prime ideal (1-prime ideal, 0-prime ideal) of R if and only if $R\backslash\theta(P) = (R\backslash\underline{\theta}(P), R\backslash\overline{\theta}(P))$ is a rough m_2 −system (m_1 −system, m_0 −system) of R.*

Proof. Assume that $\theta(P) = (\underline{\theta}(P), \overline{\theta}(P))$ is a rough 2-prime ideal of R. Let $a, b \in R\backslash\underline{\theta}(P)$. Suppose $\langle a \rangle_k . \langle b \rangle_k \subseteq \underline{\theta}(P)$. since $\underline{\theta}(P)$ is a 2-prime ideal, we have $\langle a \rangle_k \subseteq \underline{\theta}(P)$ or $\langle b \rangle_k \subseteq \underline{\theta}(P)$. Therefore $a \in \underline{\theta}(P)$ or $b \in \underline{\theta}(P)$ which is a contradiction to our assumption. Hence there exist $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $a_1 b_1 \notin \underline{\theta}(P)$. Thus $R\backslash\underline{\theta}(P)$ is an m_2 −system of R. Similarly $R\backslash\overline{\theta}(P)$ is an m_2 −system of R. Thus $R\backslash\theta(P)$ is rough m_2 −system of R.

Conversely, assume that $R\backslash\theta(P)$ is a rough m_2 −system. Suppose $\underline{\theta}(A)$ and $\underline{\theta}(B)$ are k-ideals of R such that $\underline{\theta}(A).\underline{\theta}(B) \subseteq \underline{\theta}(P)$. If $\underline{\theta}(A) \nsubseteq \underline{\theta}(P)$ and $\underline{\theta}(B) \nsubseteq \underline{\theta}(P)$, then there exist $a, b \in R$ with $a \in \underline{\theta}(A)$ and $b \in \underline{\theta}(B)$ and $a, b \notin \underline{\theta}(P)$. Therefore $a, b \in R\backslash\underline{\theta}(P)$ and since $R\backslash\underline{\theta}(P)$ is an m_2 −system, there exists $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $a_1 b_1 \in R\backslash\underline{\theta}(P)$.
Now $\langle a \rangle_k . \langle b \rangle_k \subseteq \underline{\theta}(A).\underline{\theta}(B) \subseteq \underline{\theta}(P)$.
This implies that $a_1 b_1 \in \langle a \rangle_k . \langle b \rangle_k \subseteq \underline{\theta}(P)$, Which is a contradiction to $a_1 b_1 \notin \underline{\theta}(P)$.
Thus $\underline{\theta}(A) \subseteq \underline{\theta}(P)$ or $\underline{\theta}(B) \subseteq \underline{\theta}(P)$. This shows that $\underline{\theta}(P)$ is a 2-prime ideal of R.

Similarly it can be proved that $\overline{\theta}(P)$ is a 2-prime ideal of R. Thus $\theta(P)$ is a rough 2-prime ideal of R.

Now we prove the main theorem of this section.

Theorem 4.12. *Let θ be a congruence relation on R and let $\theta(Q)$ and $\theta(M)$ be a rough ideal and a rough m_2 −system (m_1 −system) respectively of R such that $\hat{\theta}(Q) \cap \hat{\theta}(M) = \emptyset$. Then there exists a rough 2-prime ideal (1-prime ideal) $\theta(P)$ of R such that $\theta(Q) \subseteq \theta(P)$ with $\theta(P) \cap \theta(M) = \emptyset$.*

Proof. Let $\theta(Q)$ be a rough ideal and let $\theta(M)$ be a rough m_2 −system of R such that $\hat{\theta}(Q) \cap \hat{\theta}(M) = \emptyset$. Now we consider the set
$\mathcal{M} = \left\{\overline{\theta}(I) | \overline{\theta}(I) \text{is an ideal of } R \text{ such that } \overline{\theta}(Q) \subseteq \overline{\theta}(I) \text{ and } \hat{\overline{\theta}}(I) \cap \overline{\theta}(M) = \emptyset\right\}$. Clearly

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November-2014

$\overline{\theta}(Q) \in \mathcal{M}$. Let $\overline{\theta}(Q_1) \subseteq \overline{\theta}(Q_2) \subseteq \cdots$ be any chain of ideals of $\mathcal{M}$. Let $\overline{\theta}(A) = \cup\, \overline{\theta}(Q_i)$. Clearly $\overline{\theta}(A)$ is an ideal of R. Suppose that $a \in \hat{\overline{\theta}}(A) \cap \overline{\theta}(M)$, then $a \in \hat{\overline{\theta}}(A)$ and $a \in \overline{\theta}(M)$.
Hence $a + x \in \overline{\theta}(A)$ for some $x \in \overline{\theta}(A)$. Since $a + x \in \overline{\theta}(A) = \cup\, \overline{\theta}(Q_i)$, $a + x \in \overline{\theta}(Q_i)$ for some i. Since $x \in \overline{\theta}(A), x \in \overline{\theta}(Q_k)$ for some k. Without loss of generality let us assume that $k < i$. Then $a, a + x \in \overline{\theta}(Q_i)$ implies $a \in \hat{\overline{\theta}}(Q_i)$. Therefore $a \in \hat{\overline{\theta}}(Q_i) \cap \overline{\theta}(M)$ which is a contradiction since $\hat{\overline{\theta}}(Q_i) \cap \overline{\theta}(M) = \emptyset$. Thus $\hat{\overline{\theta}}(A) \cap \overline{\theta}(M) = \emptyset$. Then by Zorn's lemma there exits an ideal $\overline{\theta}(P)$ of R that is maximal with respect to the above properties. Let $\overline{\theta}(A)$ be a k-ideal of R such that $\overline{\theta}(A) \nsubseteq \overline{\theta}(P)$. Since $\overline{\theta}(P)$ is an ideal we have $\overline{\theta}(A).\overline{\theta}(P) \subseteq \overline{\theta}(P) \subseteq \hat{\overline{\theta}}(P)$ and so $\overline{\theta}(P) \subseteq \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$.

Similarly $\overline{\theta}(P) \subseteq \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l$. By Theorems 3.14 and 3.13 $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l$ and $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$ are k-ideals. Thus $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$ and $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l$ are k-ideals of R containing $\overline{\theta}(P)$.

By the maximality of $\overline{\theta}(P)$ either $\overline{\theta}(P) = \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l$ or

$\left(\widehat{\hat{\overline{\theta}}(P) : \overline{\theta}(A)}\right)_l \cap \overline{\theta}(M) = \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l \cap \overline{\theta}(M) = \emptyset$.

Similarly $\overline{\theta}(P) = \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$ or $\left(\widehat{\hat{\overline{\theta}}(P) : \overline{\theta}(A)}\right)_r \cap \overline{\theta}(M) = \left(\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r\right) \cap \overline{\theta}(M) = \emptyset$.

Suppose that $\left(\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r\right) \cap \overline{\theta}(M) = \emptyset$. Let $x \in \left(\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r\right) \cap \overline{\theta}(M)$.

Case (i): $\overline{\theta}(A) \cap \overline{\theta}(M) = \emptyset$. Let $a \in \overline{\theta}(A) \cap \overline{\theta}(M)$. Since $x \in \left(\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r\right)$, we have $\langle x \rangle_k \subseteq \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$, because $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$ is a k-ideal of R. Hence $\overline{\theta}(A)\langle x \rangle_k \subseteq \hat{\overline{\theta}}(P)$. since $a, x \in \overline{\theta}(M)$ and $\overline{\theta}(M)$ is an m_2 −system of R there exists $a_1 \in \langle a \rangle_k$ and there exists $x_1 \in \langle x \rangle_k$ such that $a_1 x_1 \in \overline{\theta}(M)$. Since $\overline{\theta}(A)\langle x \rangle_k \subseteq \hat{\overline{\theta}}(P)$ we have $a_1 x_1 \in \hat{\overline{\theta}}(P)$.
Therefore $a_1 x_1 \in \hat{\overline{\theta}}(P) \cap \overline{\theta}(M)$ which impossible as $\hat{\overline{\theta}}(P) \cap \overline{\theta}(M) = \emptyset$.
Thus $\overline{\theta}(P) = \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r$ in this case . Similarly if $y \in \left(\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r\right) \cap \overline{\theta}(M)$, then it follows that $\overline{\theta}(M) = \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l$. Thus if $\overline{\theta}(A) \cap \overline{\theta}(M) = \emptyset$, $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r = \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l = \overline{\theta}(P)$.

Case(ii): Suppose $\overline{\theta}(A) \cap \overline{\theta}(M) = \emptyset$. we have $\overline{\theta}(A)\langle x \rangle_k \subseteq \hat{\overline{\theta}}(P)$. This implies that

$\overline{\theta}(A) \subseteq \left(\hat{\overline{\theta}}(P) : \langle x \rangle_k\right)_l = \overline{\theta}(P)$ by case(i). This contradicts our assumption that $\overline{\theta}(A) \nsubseteq \overline{\theta}(P)$. Thus $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_l = \overline{\theta}(P)$ in this case . Similarly $\left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r = \overline{\theta}(P)$. Now we willshow that $\overline{\theta}(P)$ is a 2-prime ideal of R. Let $\overline{\theta}(A)$and $\overline{\theta}(B)$ be k-ideals of R such that $\overline{\theta}(A)\overline{\theta}(B) \subseteq \overline{\theta}(P)$ and $\overline{\theta}(A) \nsubseteq \overline{\theta}(P)$. Clearly $\overline{\theta}(P) \subseteq \hat{\overline{\theta}}(P)$. Hence $\overline{\theta}(A)\overline{\theta}(B) \subseteq \hat{\overline{\theta}}(P)$. This means that $\overline{\theta}(B) \subseteq \left(\hat{\overline{\theta}}(P) : \overline{\theta}(A)\right)_r = \overline{\theta}(P)$. Thus $\overline{\theta}(B) \subseteq \overline{\theta}(P)$. Hence $\overline{\theta}(P)$ is a 2-prime ideal of R.

ISSN: 2235-767X
Volume 02 Issue 11
November-2014

Adopting a similar argument on the lower approximation we can conclude that $\underline{\theta}(P)$ is a 2-prime ideal of R such that $\underline{\theta}(Q) \subseteq \underline{\theta}(P)$ with $\underline{\theta}(P) \cap \underline{\theta}(M) = \emptyset$. This shows that $\theta(P) = (\underline{\theta}(P), \overline{\theta}(P)$ is a rough 2-prime ideal of R such that $\theta(Q) \subseteq \theta(P)$ with $\theta(P) \cap \theta(M) = \emptyset$.

Theorem 4.13. *Let θ be a congruence relation in R and let P be a subset of R.*

(i) *$\overline{\theta}(P)$ is a 2-prime ideal (1-prime ideal, 0-prime ideal) of R if and only if $R\backslash\overline{\theta}(P)$ is an m_2 −system (m_1 −system, m_0 −system) of R.*

(ii) *$\underline{\theta}(P)$ is a 2-prime ideal (1-prime ideal, 0-prime ideal) of R if and only if $R\backslash\underline{\theta}(P)$ is $\underline{\theta}(P)$ is an m_2 −system (m_1 −system, m_0 −system) of R.*

Proof. (i) Assume that $\overline{\theta}(P)$ is a 2-prime ideal of R. Let $a, b \in R\backslash\overline{\theta}(P)$. Suppose $\langle a\rangle_k.\langle b\rangle_k \subseteq \overline{\theta}(P)$. Since $\overline{\theta}(P)$ is a 2-prime ideal , we have $\langle a\rangle_k \subseteq \overline{\theta}(P)$ or $\langle b\rangle_k \subseteq \overline{\theta}(P)$. Therefore $a \in \overline{\theta}(P)$ or $b \in \overline{\theta}(P)$ which is a contradiction. Hence $\langle a\rangle_k.\langle b\rangle_k \nsubseteq \overline{\theta}(P)$. Hence thereexists $a_1 \in \langle a\rangle_k$ and $b_1 \in \langle b\rangle_k$ such that $a_1 b_1 \notin \overline{\theta}(P)$. Thus $R\backslash\overline{\theta}(P)$ is an m_2 −system of R.

Conversely, let us assume that $R\backslash\overline{\theta}(P)$ m_2 −system. suppose $\overline{\theta}(A)$ and $\overline{\theta}(B)$ are k-ideals of R such that $\overline{\theta}(A)\overline{\theta}(B) \subseteq \overline{\theta}(P)$. Suppose that $\overline{\theta}(A) \nsubseteq \overline{\theta}(P)$ and $\overline{\theta}(B) \nsubseteq \overline{\theta}(P)$.

There exists $a, b \in R$ with $a \in \overline{\theta}(A)$ and $b \in \overline{\theta}(B)$ and $a, b \notin \overline{\theta}(P)$. Thus $a, b \in R\backslash\overline{\theta}(P)$ and since $R\backslash\overline{\theta}(P)$ is m_2 −system , there exist$a_1 \in \langle a\rangle_k$ and $b_1 \in \langle b\rangle_k$ such that $a_1 b_1 \in R\backslash\overline{\theta}(P)$.

Now $\langle a\rangle_k.\langle b\rangle_k \subseteq \overline{\theta}(A).\overline{\theta}(B) \subseteq \overline{\theta}(P)$. This implies that $a_1 b_1 \in \langle a\rangle_k.\langle b\rangle_k \subseteq \overline{\theta}(P)$ which is a contradiction to $a_1 b_1 \notin \overline{\theta}(P)$. Therefore $\overline{\theta}(A) \subseteq \overline{\theta}(P)$ or $\overline{\theta}(B) \subseteq \overline{\theta}(P)$.

(ii)It is proved in a similar way.

5. CONCLUSION

The theory of semirings has wide applications in several areas such as optimization theory, discrete event dynamical systems, automata theory, formal language theory and parallel computing. This paper presents the concept of $2-(0-or\ 1-)$ prime ideals in semiring and $m_2-(m_0-\ or\ m_1-)$ system in semirings through rough set theory. This work is extended to rings.

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