

Reverse Chain Rule: Study Specific to Integration by Substitution Using the Chain Rule in Reverse

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ABSTRACT

In this paper, an numerical function is considered and included the function with some particular confines to obtain the area. This study focuses about reverse chain rule specific to integration by substitution using the chain rule in reverse.

KEYWORDS: Integration, chain rule, Function, Reverse etc.

1. INTRODUCTION

We begin with a given capacity, $f(x)$ say, and ask what capacity or capacities, $F(x)$, would have $f(x)$ as their subordinate. This leads us to the ideas of an antiderivative and mix.

A prior unit, Integration as summation, clarified mix as the way toward including rectangular zones. To assess integrals defined along these lines it was important to compute the cutoff of an aggregate - a procedure which is unwieldy and unfeasible. In this unit we will perceive how integrals can be found by turning around the procedure of differentiation - that is by finding anti-derivatives.

(The Chain Rule is now and again called the Composite Functions Rule or Function of a Function Rule. In the event that we watch deliberately the appropriate responses we get when we utilize the chain run, we can figure out how to perceive when a capacity has this shape, thus find how to coordinate such capacities. Keep in mind that, if $y = f(u)$ and $u = g(x)$

So that $y = f(g(x))$, (a composite function), then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Using function notation, this can be written as $\frac{dy}{dx} = f'(g(x)) \cdot g'(x).$

In this expression, $f(g(x))$ is another way of writing $\frac{dy}{du}$ where $y = f(u)$ and $u = g(x)$ and $g(x)$ is another

way of writing $\frac{dy}{du}$ where $u = g(x)$.

Example – Find $\int 2x\sqrt{x^2+1} dx$.

Here we let $g(x) = x^2 + 1$. We have $\int 2x\sqrt{x^2+1} dx = \int \sqrt{g(x)}g'(x) dx$.

Now we let $u = g(x) = x^2 + 1$, giving us that $\frac{du}{dx} = 2x$, giving us that $du = 2x dx$. Therefore, we have

$$\int 2x\sqrt{x^2+1} dx = \int \sqrt{u} du = \frac{2u^{3/2}}{3} + C.$$

We convert our answer back to an answer in terms of the variable x , to get

$$\int 2x\sqrt{x^2+1} dx = \frac{2u^{3/2}}{3} + C = \frac{2(x^2+1)^{3/2}}{3} + C.$$

$$2x\sqrt{x^2 + 1}.$$

You should check that this the general antiderivative for by differentiating it using the chain rule.

2. REVERSING THE CHAIN RULE

In differentiation the chain rule is used to get the derivative of a composite function. For example

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

Integration is the reverse of differentiation. Therefore, we could reverse this statement and write:

$$\int 2e^{2x} dx = e^{2x}$$

(Ignore the constant for a moment). Dividing both sides by 2 we find

$$\int e^{2x} dx = \frac{e^{2x}}{2}$$

Similarly, consider $d/dx [\sin(3x+2)] = 3\cos(3x+2)$. Reversing this process:

$$\int 3\cos(3x+2) dx = \sin(3x+2)$$

Therefore:

$$\int \cos(3x+2) dx = \frac{\sin(3x+2)}{3}$$

3. REVERSE CHAIN RULE TO INTEGRATE

The reverse chain rule to integrate $\int \frac{1}{(3-5x)^4} dx$

First, write the integral in a more suitable form

$$\int (3-5x)^{-4} dx$$

Now integrate. Use the rule for integrating powers (in this case, like x^{-4}); 'Add 1 to the power. Divide by the new power and divide by the derivative of the inner function'.

4. WHEN CAN REVERSE CHAIN RULE BE USED?

The reverse chain rule can only be used when the inner function is linear, which means it has the form $mx+c$ (where m and c are constants).

Note: We cannot use the reverse chain rule when the inner function is NOT LINEAR!

$$\int (1+x^2)^5 dx$$

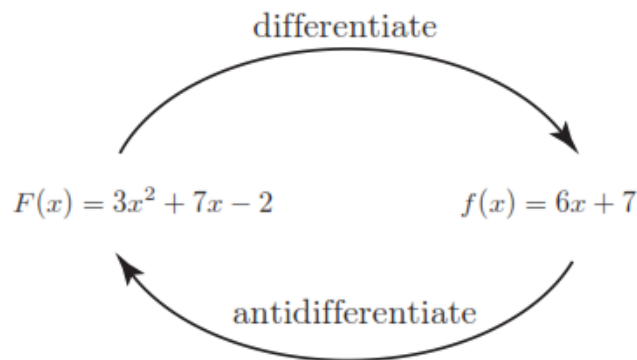
For example, we could not use it to find because the inner function $1+x^2$ is not linear.

Therefore,

$$\int (1+x^2)^5 dx \neq \frac{(1+x^2)^6}{6(2x)} + C$$

5. ANTI-DERIVATIVES - DIFFERENTIATION IN REVERSE

Consider the function $F(x) = 3x^2 + 7x - 2$. Suppose we write its derivative as $f(x)$, that is $f(x) = \frac{dF}{dx}$. You already know how to find this derivative by differentiating term by term to obtain $f(x) = \frac{dF}{dx} = 6x + 7$. This process is illustrated as-



The function $F(x)$ is an antiderivative of $f(x)$

CONCLUSION & DISCUSSION

Now assume that we work back to front and ask ourselves which function/functions could possibly have $6x+7$ as a derivative. Clearly, one answer to this question is the function $3x^2+7x-2$. We say that $F(x) = 3x^2 + 7x - 2$ is an anti-derivative of $f(x) = 6x + 7$. There is however other function which have derivative $6x + 7$. Some of these are $3x^2 + 7x + 3$, $3x^2 + 7x$, $3x^2 + 7x - 11$. The cause why all of these function has the same derivative is that the constant term disappears during differentiation. So, all of these are anti-derivatives of $6x+7$. Given any antiderivative of $f(x)$, all others can be obtained by simply adding a different constant.

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